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RESEARCH MEMORANDUM

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AN EXERCISE IN WELFARE ECONOMICS (I)

by

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An Exercise in Welfare Economics I

Table of Contents

Note that Par. I.A.2 together with the evaluation of Paragraph I.B.3 should be considered as an abstract of the present paper.

An Exercise in Welfare Economics I

Par. I.A.1. Introduction

One of the most important topics in literature on welfare economics is the empirical search for the main determinants of the outcomes of the interrelated social-economic and political processes and the desirability to maintain them or not.
In particular "classical" welfare research devotes a great deal of attention to the investigation of a social preference function, which is supposed to be maximized by a policy decision making unit and operating on an urban, regional, national or international level. Within this line of thinking, these decision units should aim ultimately at the maximization of social welfare of the community, whatever this may be. Practical "welfare" research-workers often suggest that there is enough empirical evidence for the implicit or explicit existence of a preference function, which has been used for a more or less deliberately balanced set of economic-political measures. Of course, the measures have been taken within the framework of the side-conditions imposed by the concrete structure of the economic-political system at hand.
The aforestanding postulates are translated in an econometric way by the use of a coupling of econometric models with empirically specified preference functions in order to dispose of operational decision models. However, many efforts have to be undertaken in order to overcome, or to bypass the overwhelming technical problems and to get rid of the empirical determination of the preferences of the policy decision unit and its transformation into a scalar preference function.
During the last decade we made such efforts off and on and tried to explicate implicit social preference functions for Belgium, France and the Netherlands.[1)]
For this research we introduced our so-called Deterministic-Static-Implicit-Determination-Model (DSID-Model) as a method of specifying a social preference function numerically.
Starting with a quadratic preference function (and after that with Cobb-Douglas-type preference functions) in combination with a system of

transformation functions we derived a system of equations expressing the general first-order conditions for an optimum of quantitative economic policy. Because the latter system is always underdetermined in the unknowns, we tried to get rid of this difficulty by the use of a generalized inverse after having normalized that system. Ultimately, our DSID-method chooses in the hyperplane representing the degrees of freedom, a point of minimum norm representing a Least-Least-Squares-Solution vector, of which elements are the numerically determined parameters of an a priori specified preference function (see also paragraph I.B.2).
However, our aforementioned applications to the cases of Belgium, France and the Netherlands still left a number of problems unsolved.[2] The first one is that the results were dominantly influenced by the functional form and the arguments contained in both the preference function and the side model. The second problem is of a more general nature viz. that of the second order conditions. One way how the latter problems could be side-stepped is to consider the trends only in the relative preference elasticities of a policy decision unit. If we accept the issue of the relative preference elasticity (see I.B.2) and its evolution over time as indicative of the evolution of implicit policy unit preferences then we do not have to be bothered about the functional form of the preference function. This is because it can be shown that there always exists a function that will be maximized (see Appendix A).
The third problem is the most fundamental problem for econometrics as a science: "How should econometrically founded decision models isolate in a correct way the variables (and their numerical realizations in the past) with regard to the <u>true</u> economic-political measures of the policy-decision unit?" An elegant way to overcome this latter problem may be if we agreed to consider it for the time being in the laboratory of economic model-building (see paragraph I.B.1).

In order to explicate the aforestated problems in a more explicit way, we intend to publish two research-memoranda of which the paper at issue is the first one.

Par. I.A.2. The program of the paper

More or less in the laboratory of "roughly empirical" model-building in "Tilburg"-style we develop in par. I.B.1 our ideas about the functioning of an open economic system and establish a linearly specified model. Although the model permits a large measure of flexibility for the analysis of different economic systems especially with respect to demand or supply dominancy at the different markets to be distinguished, we confine ourselves to the case of demand dominance at two markets.
For the sake of exercise in par. I.B.2 we can use this model for the generation of ex post numerical results of the endogenous variables of an economic system in consequence of economic political measures that have been taken by a policy decision unit. Besides the model allows for a correct and explicit specification of these measures shaping them by the variables of automous pushes and pulls. After the analysis of the dynamic properties of our "conjunctural-structural" (CS)-model in economic theoretical terms we can start in par. I.B.2 with our demonstration how we can derive some important characteristics of the preference structure underlying a preference function on which optimal economic policy is assumed to be performed in the past.
After the formal explication of the assumed optimal quantitative economic policy problem we will explain the main elements of our DSID-model idea by:

1. establishing the Lagrange function;
2. assuming ex post knowledge of the numerical values of target and instrumental variables;
3. using the Moore-Penrose inverse of a matrix and its numerical computation;
4. deriving the relative preferences and relative preference elasticity ratios corresponding to different observation horizons;
5. investigating on stability through time of the listed relative preference elasticity data making use of "orthogonal polynomial fitting" and of the "analysis of variance";
6. a graphical shaping of the original DSID-model results with regard to the Real Gross National Product-variable (as an illustration).

The paper will be concluded by an evaluation of the results and with notes and bibliographical references in par. I.B.3.
Finally a proof of existence of the global maximum of an objective function has been given in Appendix A.

Par. I.B.1. The 'CS'-model for an open economy

In this paragraph we shall develop our ideas about the functioning of an open economic system including a labour market and a product market. At this stage we do not have accurate knowledge about the preference structure underlying a preference function on which optimal economic policy is assumed to be performed by the policy decision unit of this economy. The model to be developed will enable us to evaluate short run as well as long run consequences of economic-political measures without questioning whether they are to be conceived of as optimal or not.
For the sake of the exercise in next paragraph we use the pure demand model idea with respect to the product market, but the labour market operates under a more modest imperfect regime. The marketclearing function of prices on the product market is taken over by the rate of underutilization of national production capacity whereas the marketclearing function of wages on the labour market can partly do his job and otherwise is partly taken over by the rate of unemployment.[3)]

Note, however, that the model permits a large measure of flexibility for the analysis of the consequences of demand or supply dominance at the two markets, depending on the desired particular situations at hand.

In order to avoid problems of non-linear model specification we shall formulate the variables in our model in (percentage) terms of relative differences with respect to their path of exponential or equilibrium growth except for those of the balance of exports and imports and the rate of utilization of national production capacity. Their relative differences are expressed in terms of the equilibrium value of the Gross National Product.
The symbols that stand for absolute values are denoted by '~', and the subscript '0' relates to the equilibrium values of the variables. Without this subscript they relate to their actual values.
Exogenous variables have a dash underneath. So we define, $\tilde{x}$ being the real or actual value and $\tilde{x}_0$ being the corresponding equilibrium value of a variable, the relative (percentage) difference of x of the same variable in period t as:

$$x_t = \frac{\tilde{x}_t - \tilde{x}_{0_t}}{\tilde{x}_{0_t}} \cdot 100$$

In the equilibrium neighbourhood the difference of x_t is approximately equal to the extra rate of growth of this variable with respect to the preceding period[4], i.e.:

$$\Delta x_t = x_t - x_{t-1} \simeq \{\frac{\tilde{x}_t - \tilde{x}_{t-1}}{\tilde{x}_{t-1}} - \frac{\tilde{x}_{0_t} - \tilde{x}_{0_{t-1}}}{\tilde{x}_{0_{t-1}}}\} \cdot 100$$

Now we can formulate <u>the model</u>

1. National employment is determined by the national capital stock (= national production capacity):

$$\ell_t = k_t \tag{I.B.1a}$$

2. The national capital stock is influenced by the effect of relative differences in investments yesterday:

$$\Delta k_t = \frac{\sigma}{\kappa} (i_{t-1} - k_{t-1}) \tag{I.B.1b}$$

3. Real investment today is determined by the national profit income share today and the volume of the national capital stock today:

$$i_t = \frac{\tilde{\sigma}_R}{\sigma} Y'_{R_t} + k_t \tag{I.B.1c}$$

4. Definition of national profit income share:

$$Y'_{R_t} \equiv - \frac{\lambda}{1-\lambda} (w'_t) \tag{I.B.1d}$$

5. Definition of national labour income share:

$$w'_t \equiv Y_{L_t} - Y_{i_t} \tag{I.B.1e}$$

6. Definition of national income:

$$Y_{i_t} \equiv Y_{p_t} + \mu_0 \, (p_{e_t} - p_{m_t} - p_{w_t}) \qquad \text{(I.B.1f)}$$

7. Definition of labour income:

$$Y_{L_t} \equiv \ell_t + w_t \qquad \text{(I.B.1g)}$$

8. The real wage rate per worker today is determined by the level of national employment today and by an exogenous wage push:

$$w_t = \beta \ell_t + \underline{p\ell}_t \qquad \text{(I.B.1h)}$$

9. Definition of national production:

$$Y_{p_t} \equiv x_t + s_{u_t} \qquad \text{(I.B.1i)}$$

10. The balance of exports and imports is determined by the level of national real spendings and the real exchange rate:

$$s_{u_t} = -\tilde{\mu} \, x_t - \mu_0 \eta \, (p_{e_t} - p_{m_t} - p_{w_t}) \qquad \text{(I.B.1j)}$$

11. National real spendings equal national real consumption and national real investment:

$$x_t \equiv (1-\sigma) c_t + \sigma \, i_t \qquad \text{(I.B.1k)}$$

12. National real consumption is determined by national labour income and partially derives out of profit income; moreover it is influenced by an exogenous consumption pull:

$$c_t = \tilde{\gamma}'_L \, Y_{L_t} + \tilde{\gamma}'_R \, w_t + \frac{1}{1-\sigma} \, \underline{x}_t \qquad \text{(I.B.1l)}$$

13. Definition of the real wage rate per worker:

$$w_t \equiv p\ell_t - p_t \qquad \text{(I.B.1m)}$$

14. The nominal exchange rate and the import price level are fixed on their equilibrium level; the export price level equals the internal price level:

$$p_{w_t} = 0 \;; \quad p_{m_t} = 0 \quad \text{and} \quad p_{e_t} = p_t \tag{I.B.1n}$$

15. Definition of the rate of utilization of national production capacity in terms of equilibrium capacity:

$$-s_{b_t} \equiv y_{p_t} - k_t \tag{I.B.1o}$$

16. The internal price level today is determined by the nominal wage rate per worker and the import price level yesterday; moreover by an exogenous price push:

$$p_t = (1-\mu)p\ell_{t-1} + \mu(p_{m_{t-1}} + p_{w_{t-1}}) + \underline{p}_t \tag{I.B.1p}$$

Greek symbols stand for the following ratios and parameters:

$\sigma = \tilde{i}_0/\tilde{y}_0$	= investment-output ratio
$\kappa = \tilde{k}_0/\tilde{y}_0$	= capital-output ratio
$\lambda = \tilde{y}_{L_0}/\tilde{y}_0$	= labour share of national income
$1-\lambda = \tilde{y}_{R_0}/\tilde{y}_0$	= capital share of national income
β	= elasticity of real wages with respect to national employment
$\dfrac{\tilde{\mu}}{\mu_0}$	= import elasticity with respect to national spendings
$\dfrac{\tilde{\sigma}_R}{\sigma}$	= investment elasticity with respect to the national profit income share

$\tilde{\gamma}'_L$ = consumption elasticity with respect to labour income

$\tilde{\gamma}'_R$ = consumption (by the profit earners) elasticity with respect to the real wage level

η = sum of import and export elasticities with respect to the real exchange rate

$\mu_0 = \tilde{e}_0/\tilde{y}_0 = \tilde{m}_0/\tilde{y}_0$ = export and import shares of total demand (= national production)

μ = import price share of price formation

$(1-\mu)$ = labour cost share of price formation

The model has 18 equations and 18 unknowns (apart from the pushes and pulls) and thus is determined.
In the context of this exercise our special attention is turned to the variables with respect to the profit share of national income y'_{R_t}, national production y_{p_t}, the balance of exports and imports s_{u_t}, national labour income y_{L_t}, national spendings x_t and the internal price level p_t.
Hereafter we shall formulate three final equations of the model in terms of the labour income, the national spendings and the internal price level variables and the relevant autonomous pushes and pulls. We can use them for the analysis of the stability characteristics and the economic-theoretical implications of the integral model. Moreover, they enable us to calculate the numerical consequences for the other three variables caused by economic-political measures which are shaped now as:

1. An incidental, but not permanent positive wage push, i.e.
   $\underline{p\ell}_t = 1.44 \;\forall\; t \geq 1$.
2. A permanent positive internal spending pull, i.e.,
   $\underline{x}_t = 9.00 \;\forall\; t \geq 1$.

3. An incidental, but not permanent positive internal price push, i.e., $\underline{p}_t = 3.00 \ \forall \ t \geq 1$.

In this special case we make the choice for the numerical values of ratios and parameters as follows:

$\sigma = 1/4 \qquad \kappa = 2 \qquad \lambda = 1/2 \qquad \beta = 2 \qquad \tilde{\mu} = 1/2 \qquad \mu_0 = 1/2$

$\tilde{\sigma}_R = 1/2 \qquad \tilde{\gamma}'_L = 2/3 \qquad \tilde{\gamma}'_R = 1/6 \qquad \eta = 4\tfrac{1}{2} \qquad \mu = 1/3$

By means of substitution the reduced forms to be derived are:

$$y_{L,t} - y_{L,t-1} + y_{L,t-2} = \underline{p}_{\ell,t} - \frac{27}{16}\underline{p}_{\ell,t-1} + \frac{7}{24}\underline{p}_{\ell,t-2} + \frac{1}{2}\underline{x}_{t-1} - \frac{1}{3}\underline{x}_{t-2} - \frac{7}{4}\underline{p}_{t-1} \qquad \text{(I.B.1q)}$$

$$x_t - x_{t-1} + x_{t-2} = \frac{1}{6}\underline{p}_{\ell,t} - \frac{79}{72}\underline{p}_{\ell,t-1} + \frac{5}{8}\underline{p}_{\ell,t-2} + \frac{4}{3}\underline{x}_t - \frac{11}{9}\underline{x}_{t-1} + \underline{x}_{t-2} - \frac{7}{6}\underline{p}_t \qquad \text{(I.B.1r)}$$

$$p_t - p_{t-1} + p_{t-2} = \frac{2}{3}\underline{p}_{\ell,t-1} - \frac{19}{36}\underline{p}_{\ell,t-2} + \frac{2}{9}\underline{x}_{t-2} + \underline{p}_t - \frac{1}{3}\underline{p}_{t-1} \qquad \text{(I.B.1s)}$$

It is note-worthy that these three reduced forms have the same characteristic equations which are common to all of the endogenous variables of the model. Hence each of them is bound in the linear difference equation system of our model and has the same characteristic roots and by this all of them have the same inherent stability, periodicity and amplitude (damping characteristics).

From our numerically specified characteristic equations with regard to (I.B.1q)/(I.B.1s) it is easy to verify that we are dealing with a regular business cycle movement with a periodicity of six years and an amplitude value of $(1)^t = 1$.

However, the particular response time path of any endogenous variable does not depend alone on the mentioned inherent response characteristics of the system. It is extra determined by the particular sequence of the

pushes and pulls beginning from a particular initial situation. This circumstance causes the possibility of a different particular response time path for each variable (see tables I.B.1.1/I.B.1.3).
Again, it is note-worthy that in our special case the exogenous stimuli become constant over time after a particular adaptive initial situation which results if the pushes or pulls start to operate as autonomously impressed forces in period 1, caused by the policy decision unit. In mathematical terms it means that the particular solution for any endogenous variable, too, is constant and may be identified as a new equilibrium or trend solution. In the tables I.B.1.1/I.B.1.3 it is of course formulated in relative terms with respect to the original equilibrium growth path solution of such a variable.

The analysis of the dynamic properties of our CS-model in economic-theoretical terms can be more adequately performed by evaluation of the numerical results summarized in the tables I.B.1.1/I.B.1.3.
Each of the three tables shows the consequences of a push or pull (the last row) with respect to the three target variables $y'_{R,t}$, $y_{p,t}$ and $s_{u,t}$ (the first three rows) respectively to the three instrumental variables $y_{L,t}$, x_t and p_t (the intermediate three rows).
For any variable the relative initial situation is, of course, a zero situation (the first column). The columns 1 to 7 indicate the periodical results for each variable after the moment that the relevant push or pull is realized.
Adding up the values of six of these columns and again dividing them by six, will yield the trend value of a variable as indicated in the most right-side column of a table.

Although the mentioned three tables only review the particular responses of a variable to one of the three individual autonomous impulses, it is easy to understand how the corresponding results of simultaneous combinations of these impulses can be calculated. For linearity of our CS-model means that a simple adding-up procedure must be performed on the same rows in two or three tables with regard to one and the same variable. For the sake of demonstration in next paragraph we actually did for different combinations of trend-results (see columns $\bar{t}$ = 1 + 2 and $\bar{t}$ = 1 + 2 + 3 of table I.B.2.3).

Table I.B.1.1.

Autonomous Impuls	Incidental, but not permanent, nominal wage push of 1.44 percent in period 1.								
Variables \ Period t =	0	1	2	3	4	5	6	7	Trend Case 1 $\triangleq \bar{t} = 1$
$y'_{R,t} = y_{p,t} + 0{,}5 p_t - y_{L,t}$	0.00	-1.32	-2.68	-1.36	+1.32	+2.68	+1.36	-1.32	$\bar{y}'_{R,\bar{t}} = 0.00$
$y_{p,t} = s_{u,t} + x_t$	0.00	0.12	-2.71	-3.50	-1.46	1.37	2.16	0.12	$\bar{y}_{p,\bar{t}} = -0.67$
$s_{u,t} = -0.5 x_t - 2.25 p_t$	0.00	-0.12	-1.61	-1.72	-0.34	1.15	1.26	-0.12	$\bar{s}_{u,\bar{t}} = -0.23$
$y_{L,t} = y_{L,t-1} - y_{L,t-2} + \underline{p}_{\ell,t} - \frac{81}{48}\underline{p}_{\ell,t-1} + \frac{21}{72}\underline{p}_{\ell,t-2}$	0.00	1.44	0.45	-1.56	-2.58	-1.59	0.42	1.44	$\bar{y}_{L,\bar{t}} = -\frac{19}{48}\bar{\underline{p}}_{\ell,\bar{t}} = -0.57$
$x_t = x_{t-1} - x_{t-2} + \frac{12}{72}\underline{p}_{\ell,t} - \frac{79}{72}\underline{p}_{\ell,t-1} + \frac{45}{72}\underline{p}_{\ell,t-2}$	0.00	0.24	-1.10	-1.78	-1.12	0.22	0.90	0.24	$\bar{x}_{\bar{t}} = -\frac{22}{72}\bar{\underline{p}}_{\ell,\bar{t}} = -0.44$
$p_t = p_{t-1} - p_{t-2} + \frac{2}{3}\underline{p}_{\ell,t-1} - \frac{19}{36}\underline{p}_{\ell,t-2}$	0.00	0.00	0.96	1.16	0.40	-0.56	-0.76	0.00	$\bar{p}_{\bar{t}} = \frac{5}{36}\bar{\underline{p}}_{\ell,\bar{t}} = 0.20$
$\underline{p}_{\ell,t}$	0.00	1.44	1.44	1.44	1.44	1.44	1.44	1.44	$\bar{\underline{p}}_{\ell,\bar{t}} = 1.44$

Table I.B.1.2.

Autonomous Impuls	Permanent internal spending pull of 9.00 percent at the beginning of period 1.								
Period t = / Variables	0	1	2	3	4	5	6	7	Trend Case 2 $\triangleq \bar{t} = 2$
$y'_{R,t} = y_{p,t} + 0.5p_t - y_{L,t}$	0.00	6.00	2.00	- 4.00	- 6.00	- 2.00	4.00	6.00	$\bar{y}'_{R,\bar{t}} = 0.00$
$y_{p,t} = s_{u,t} + x_t$	0.00	6.00	6.50	1.00	- 5.00	- 5.50	0.00	6.00	$\bar{y}_{p,\bar{t}} = 0.50$
$s_{u,t} = -0.5x_t - 2.25p_t$	0.00	-6.00	-6.50	-10.00	-13.00	-12.50	-9.00	-6.00	$\bar{s}_{u,\bar{t}} = -9.50$
$y_{L,t} = y_{L,t-1} - y_{L,t-2} + \frac{1}{2}\underline{x}_{t-1} - \frac{1}{3}\underline{x}_{t-2}$	0.00	0.00	4.50	6.00	+ 3.00	- 1.50	-3.00	0.00	$\bar{y}_{L,\bar{t}} = \frac{1}{6}\bar{\underline{x}}_{\bar{t}} = 1.50$
$x_t = x_{t-1} - x_{t-2} + \frac{4}{3}\underline{x}_t - \frac{11}{9}\underline{x}_{t-1} + \underline{x}_{t-2}$	0.00	12.00	13.00	11.00	8.00	7.00	9.00	12.00	$\bar{x}_{\bar{t}} = \frac{10}{9}\bar{\underline{x}}_{\bar{t}} = 10.00$
$p_t = p_{t-1} - p_{t-2} + \frac{2}{9}\underline{x}_{t-2}$	0.00	0.00	0.00	2.00	4.00	4.00	2.00	0.00	$\bar{p}_{\bar{t}} = \frac{2}{9}\bar{\underline{x}}_{\bar{t}} = 2.00$
$\underline{x}_t$	0.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00	$\bar{\underline{x}}_{\bar{t}} = 9.00$

Table I.B.1.3.

Autonomous Impuls	Incidental, but not permanent internal price push of 3.00 percent in period 1.								
Period t = / Variables	0	1	2	3	4	5	6	7	Trend Case 3 $\triangleq$ $\bar{t}$ = 3
$y'_{R,t} = y_{p,t} + 0.5p_t - y_{L,t}$	0.00	-7.00	- 7.00	0.00	7.00	7.00	0.00	-7.00	$\bar{y}'_{R,\bar{t}} = 0.00$
$y_{p,t} = s_{u,t} + x_t$	0.00	-8.50	-14.75	-12.50	- 4.00	2.25	0.00	-8.50	$\bar{y}_{p,\bar{t}} = -6.25$
$s_{u,t} = -0.5x_t - 2.25p_t$	0.00	-5.00	- 7.75	- 5.50	- 0.50	2.25	0.00	-5.00	$\bar{s}_{u,\bar{t}} = -2.75$
$y_{L,t} = y_{L,t-1} - y_{L,t-2} - \frac{7}{4}\underline{p}_{t-1}$	0.00	0.00	- 5.25	-10.50	-10.50	-5.25	0.00	0.00	$\bar{y}_{L,\bar{t}} = -\frac{7}{4}\bar{\underline{p}}_{\bar{t}} = -5.25$
$x_t = x_{t-1} - x_{t-2} - \frac{7}{6}\underline{p}_t$	0.00	-3.50	- 7.00	- 7.00	- 3.50	0.00	0.00	-3.50	$\bar{x}_{\bar{t}} = -\frac{7}{6} \cdot \bar{\underline{p}}_{\bar{t}} = -3.50$
$p_t = p_{t-1} - p_{t-2} + \underline{p}_t - \frac{1}{3}\underline{p}_{t-1}$	0.00	3.00	5.00	4.00	1.00	-1.00	0.00	3.00	$\bar{p}_{\bar{t}} = \frac{2}{3}\bar{\underline{p}}_{\bar{t}} = 2.00$
$\underline{p}_t$	0.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	$\bar{\underline{p}}_{\bar{t}} = 3.00$

Let us draw shortly a few conclusions with respect to the structural effects of the mentioned autonomously impressed forces on our economic system. The business cycle conclusions to be drawn will be left to the reader.

Table I.B.1.1 deals a.o. with Trend Case 1 of an incidental but not permanent nominal wage push of 1.44 percent in period 1, from which it follows that it

- does not touch the profit income share,
- deteriorates national production because of deteriorizations of the international competitive power and the national spending power.
  Although the terms of trade have been ameliorated a little bit, the competitive power has been worsened strongly caused by shift-off processes which started to operate after the wage push was realized. These shift-off processes explain why the internal price level has been increased and the balance of exports and imports became negative.
- Finally, negative multiplier-accelerator processes can explain why national labour income as well as national spendings have been worsened in the long run.

Table I.B.1.2 deals with Trend Case 2 of a permanent internal spending pull of 9 percent from period 1 on and makes clear that

- again, it does not touch the profit income share,
- it ameliorates in a moderate way national production but deteriorates the competitive power and causes a permanent situation of excess internal spendings. By this the balance of exports and imports deteriorates enormously.
  The worsening of the competitive power as well as the amelioration of the terms of trade are again caused in consequence of shift-off processes.

Table I.B.1.3 deals with Trend Case 3 of an incidental but not permanent internal price push of 3.00 percent in period 1 and shows that

- still again, it does not affect the profit income share,
- it deteriorates the competitive power in such a way that decreasing national spendings cannot compensate the negative movement of national

production. By this the balance of exports and imports deteriorates considerably.
The negative development of the competitive power and its counter part of a positive movement of the terms of trade are caused in consequence of shift-off processes like we saw.

Last but not least it is worthwile to note that all the results, listed in the tables I.B.1.1/I.B.1.3, are consequences of positively valued pushes and pulls. The symmetrical nature of the 'CS'-model will cause the exact reverse results if these pushes and pulls had been valued in a negative way and letting these values unchanged with respect to their absolute magnitude.

Par. I.B.2. A demonstration

In this paragraph we shall explain how we can derive some important characteristics of the preference structure underlying a preference function on which optimal economic policy is assumed to be performed in the past. With the help of this simple example a demonstration can be given how the theoretical DSID-model-idea is suitably used in order to find the numerical values of relative preference elasticities during the years of our horizon of investigation.

Within the theoretical framework of the model, used in the preceding paragraph, the social welfare implications of the cyclical-structural results generated by this same 'CS'-model of a national economy, are investigated.

Three equations of the model are considered as the actually known constraints of an assumed optimal quantitative economic policy as has been formalized in scheme (I.B.2a).

Taking account of the other assumptions that we stated already in foregoing paragraph, we are able now to denote how the DSID-model procedure works.

In order to detect the preference structure which is consistent with the generated results in consequence of the mentioned wage push, internal spending pull respectively the nominal price push and combinations of these three possibilities, the assumed optimal quantitative economic policy problem can be formalized as follows:

Maximize: $\omega_t\ (y'_{R,t},\ y_{p,t},\ s_{u,t},\ y_{L,t},\ x_t,\ p_t)$

subject to: $y'_{R,t} \equiv y_{p,t} - y_{L,t} + \mu_0\ (p_{e,t} - p_{m,t})$ (I.B.2a)

$$y_{p,t} \equiv s_{u,t} + x_t$$

$$s_{u,t} = -\tilde{\mu}\ x_t - \mu_0 \eta\ (p_{e,t} - p_{m,t})$$

setting: $\mu_0 = \frac{1}{2}$; $\tilde{\mu} = \frac{1}{2}$; $p_{m,t} = 0$; $p_{e,t} = p_t\ \forall_t^{1,\ldots T}$ and $\eta = 4\frac{1}{2}$

we get for (I.B.2a):

Maximize: $\omega_t\ (y'_{R,t},\ y_{p,t},\ s_{u,t},\ y_{L,t},\ x_t,\ p_t)$

subject to: $y'_{R,t} - y_{p,t} + y_{L,t} - \frac{1}{2}\, p_t = 0$ (I.B.2b)

$y_{p,t} - s_{u,t} - x_t = 0$

$s_{u,t} + \frac{1}{2}\, x_t + \frac{9}{4}\, p_t = 0$

where $y'_{R,t}$, $y_{p,t}$ and $s_{u,t}$ are conceived of as potentially relevant target variables whereas $y_{L,t}$, x_t and p_t may be considered in this case as potentially relevant instrumental variables of economic policy.
From the Jacobian-matrix, to be derived from the constraints of (I.B.2b), it becomes clear that the conditions of Appendix A are satisfied.
Establishing the Lagrange function and taking its first partial derivatives with respect to $y'_{R,t}$, $y_{p,t}$, $s_{u,t}$, $y_{L,t}$, x_t, p_t and the Lagrange multipliers $\lambda_{i,t}$ $(i = 1,\ldots,3;\ t = 1,\ldots T)$ yields:

$$\begin{aligned} L_t = {} & \omega_t(y'_{R,t},\ y_{p,t},\ s_{u,t},\ y_{L,t},\ x_t,\ p_t) - \\ & - \lambda_{1,t}(y'_{R,t} - y_{p,t} + y_{L,t} - \tfrac{1}{2}\, p_t) - \\ & - \lambda_{2,t}(y_{p,t} - s_{u,t} - x_t) - \lambda_{3,t}(s_{u,t} + \tfrac{1}{2}\, x_t + \tfrac{9}{4}\, p_t) \end{aligned}$$

$$\frac{\delta\ L_t}{\delta\ y'_{R,t}} = \frac{\delta\omega_t}{\delta\ y'_{R,t}} - \lambda_{1,t}\cdot 1 - \lambda_{2,t}\cdot 0 - \lambda_{3,t}\cdot 0 = 0$$

$$\frac{\delta\ L_t}{\delta\ y_{p,t}} = \frac{\delta\omega_t}{\delta\ y_{p,t}} + \lambda_{1,t}\cdot 1 - \lambda_{2,t}\cdot 1 - \lambda_{3,t}\cdot 0 = 0$$

$$\frac{\delta\ L_t}{\delta\ s_{u,t}} = \frac{\delta\omega_t}{\delta\ s_{u,t}} - \lambda_{1,t}\cdot 0 + \lambda_{2,t}\cdot 1 - \lambda_{3,t}\cdot 1 = 0$$

$$\frac{\delta\ L_t}{\delta\ y_{L,t}} = \frac{\delta\omega_t}{\delta\ y_{L,t}} - \lambda_{1,t}\cdot 1 - \lambda_{2,t}\cdot 0 - \lambda_{3,t}\cdot 0 = 0$$

(I.B.2c)

$$\frac{\delta\ L_t}{\delta\ x_t} = \frac{\delta\omega_t}{\delta\ x_t} - \lambda_{1,t}\cdot 0 + \lambda_{2,t}\cdot 1 - \lambda_{3,t}\cdot\frac{1}{2} = 0$$

$$\frac{\delta\ L_t}{\delta\ p_t} = \frac{\delta\omega_t}{\delta\ p_t} + \lambda_{1,t}\cdot\frac{1}{2} - \lambda_{2,t}\cdot 0 - \lambda_{3,t}\cdot\frac{9}{4} = 0$$

$$\frac{\delta\ L_t}{\delta\lambda_{1,t}} = y'_{R,t} - y_{p,t} + y_{L,t} - \frac{1}{2}\,p_t = 0$$

$$\frac{\delta\ L_t}{\delta\lambda_{2,t}} = y_{p,t} - s_{u,t} - x_t = 0$$

$$\frac{\delta\ L_t}{\delta\lambda_{3,t}} = s_{u,t} + \frac{1}{2}\,x_t + \frac{9}{4}\,p_t = 0$$

Ex post knowledge of the numerical values of the target and instrumental variables for every year t, enables us to single out the last three equations of system (I.B.2c). Rearranging and writing this system in matrix-notation, we get:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & +1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & +1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & +1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & +\frac{1}{2} & 0 & -\frac{9}{4} \end{bmatrix} \cdot \begin{bmatrix} \omega^*_{y'_{R,t}}(y'_{R,t}, y_{p,t}, s_{u,t}, y_{L,t}, x_t, p_t) \\ \omega^*_{y_{p,t}}(y'_{R,t}, y_{p,t}, s_{u,t}, y_{L,t}, x_t, p_t) \\ \omega^*_{s_{u,t}}(y'_{R,t}, y_{p,t}, s_{u,t}, y_{L,t}, x_t, p_t) \\ \omega^*_{y_{L,t}}(y'_{R,t}, y_{p,t}, s_{u,t}, y_{L,t}, x_t, p_t) \\ \omega^*_{x_t}(y'_{R,t}, y_{p,t}, s_{u,t}, y_{L,t}, x_t, p_t) \\ \omega^*_{p_t}(y'_{R,t}, y_{p,t}, s_{u,t}, y_{L,t}, x_t, p_t) \\ \lambda_{1,t} \\ \lambda_{2,t} \\ \lambda_{3,t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{(I.B.2d)}$$

If we denote the 6×9-matrix of known elements by A_t, the 9×1-vector of unknowns by $\underline{p}'_t$ and the 6×1-nullvector by $\underline{0}$ we derive from (I.B.2d):

$$A_t \cdot \underline{p}_t' = \underline{0} \qquad \text{(I.B.2e)}$$

In this special case $A_t = A$ is a constant matrix of year t. System (I.B.2e) is underdetermined because the number of potentially feasible target variables exceeds zero. Normalizing this system by setting $\lambda_{3,t}$ equal to the arbitrary value of one we get:

$$\bar{A}_t \cdot \bar{\underline{p}}_t' = \underline{q}_t \qquad \text{(I.B.2f)}$$

where $\bar{A}_t$ is the original A_t-matrix of system (I.B.2d) without its last column and $\bar{\underline{p}}_t'$ is the unknown-vector without its last element $\lambda_{3,t} = 1$; $\underline{q}_t$ is the vector resulting from multiplication of the last column of the original matrix A_t by $\lambda_{3,t} = 1$ and inverting the sign.
System (I.B.2f) can be solved for the as yet unknown elements of the 8×1-vector $\bar{\underline{p}}_t'$ up to the arbitrary fixed base value of 1, using <u>the Moore-Penrose inverse of the matrix</u> $\bar{A}_t$ denoted by $\bar{A}_t^+$, i.e.,

$$\bar{\underline{p}}_{t,u}' = \bar{A}_t^+ \cdot \underline{q}_t \qquad \text{(I.B.2g)}$$

From (I.B.2d), (I.B.2e) and (I.B.2f) it is clear that we are dealing with a consistent system because the ranks of the matrix A_t and $\bar{A}_t$ are of same order.
Therefore, we get the following general solution of this system:

$$\min_{\bar{\underline{p}}_t'} \|\bar{A}_t \cdot \bar{\underline{p}}_t' - \underline{q}_t\|_2 = \|\bar{A}_t \cdot \bar{\underline{p}}_{t,0}' - \underline{q}_t\|_2 = 0 \qquad \text{(I.B.2h)}$$

where

$$\bar{\underline{p}}_{t,0}' = \bar{A}_t^+ \cdot \underline{q}_t + (I - \bar{A}_t^+ \bar{A}_t)\underline{r}_t \qquad \text{(I.B.2i)}$$

where $\underline{r}_t$ is an arbitrary real-valued 8×1-vector and $\bar{\underline{p}}_{t,0}'$ is the 8×1 Least Squares Solution-vector.

The unique solution satisfying the minimum Euclidean norm is found by:

$$\bar{\underline{p}}_{t,u}' = \min_{\underline{r}_t} \|\bar{\underline{p}}_{t,0}'\|_2 = \bar{A}_t^+ \cdot \underline{q}_t \qquad \text{(I.B.2j)}$$

where $\bar{p}'_{t,u}$ is the 8×1 Least Least Squares Solution-vector.

The argumentation used above can be repeated for every year t (t = 1,...,T) where we select every time a $\bar{p}'_{t,u}$-vector. The ratio-values of the first six elements of this vector expressed in terms of each other denote the numerical values of the relative (marginal) preferences of the target- and instrumental variables whatever the actual value of $\lambda_{3,t} \neq 0 \quad \forall_t^{t=1,\ldots,T}$. If we have performed the numerical computation for a number of years using the procedure explained elsewhere[5], we can test stability through time of the relative preferences and of the relative preference elasticities, where the latter can be derived from the former as indicated in the systems (I.B.2p) and (I.B.2q).

Let us turn to <u>the numerical computation of</u> $\bar{A}_t^+$ of system (I.B.2j) starting from $\bar{A}_t$ of system (I.B.2f). We have already noted that our special example yields constant A_t and $\bar{A}_t$ matrices $\forall_t^{t=1,\ldots,T}$. This circumstance considerably facilitates the computation.
The same is true because of the constancy of the $\underline{q}_t$-vector.
It means that we have to compute only one time $\bar{A}_t^+$ (t=1) and $\bar{A}_t^+ \cdot \underline{q}_t$ (t=1) and these results can be used for getting $\bar{p}'_{t,0}$ and $\bar{p}'_{t,u}$ of system (I.B.2i) resp. (I.B.2j) for t = 2,...,T.
From (I.B.2i) we derive:

$$\forall_t^{t=1,\ldots,T} \quad \bar{\underline{p}}'_{t,0} = \bar{A}_t^+ \underline{q}_t + (I-\bar{A}_t^+\bar{A}_t)\underline{r}_t = \bar{\underline{p}}'_0 = \bar{A}^+\underline{q} + (I-\bar{A}^+\bar{A})\underline{r} \tag{I.B.2k}$$

From (I.B.2j) it follows now:

$$\forall_t^{t=1,\ldots,T} \qquad \bar{\underline{p}}'_{t,u} = \min_{\underline{r}_t} \|\bar{\underline{p}}'_{t,0}\|_2 = \bar{A}_t^+ \cdot \underline{q}_t =$$

$$\bar{\underline{p}}'_u = \min_{\underline{r}} \|\bar{\underline{p}}'_0\|_2 = \bar{A}^+\underline{q} \tag{I.B.2l}$$

Using the arithmetical technique applied by Graybill[5] and starting from the matrix $\bar{A}_t = \bar{A}$ of system (I.B.2f) the final result of the first five interations yields:

$$\bar{A}_6^+ = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

whereas the seventh (and last) iteration results into:

$$\bar{A}_8^+ = \begin{bmatrix} 48/64 & 12/64 & 4/64 & -16/64 & 4/64 & 8/64 \\ 12/64 & 39/64 & 13/64 & 12/64 & 13/64 & -6/64 \\ 4/64 & 13/64 & 47/64 & 4/64 & -17/64 & -2/64 \\ -16/64 & 12/64 & 4/64 & 48/64 & 4/64 & 8/64 \\ 4/64 & 13/64 & -17/64 & 4/64 & 47/64 & -2/64 \\ 8/64 & -6/64 & -2/64 & 8/64 & -2/64 & 60/64 \\ -16/64 & 12/64 & 4/64 & -16/64 & 4/64 & 8/64 \\ -4/64 & -13/64 & 17/64 & -4/64 & 17/64 & 2/64 \end{bmatrix} \tag{I.B.2m}$$

Matrix (I.B.2m) appears to be the Moore-Penrose inverse of the matrix $\bar{A}$, satisfying the four necessary and sufficient conditions of the Moore-Penrose inverse-definition. With the help of this matrix we derive for (I.B.2k):

$$\bar{\underline{p}}'_o = \bar{A}^+ \cdot \underline{g} + (I-\bar{A}^+\bar{A})\underline{r} =$$

$$\begin{bmatrix} \bar{p}'_{o,1} \\ \bar{p}'_{o,2} \\ \bar{p}'_{o,3} \\ \bar{p}'_{o,4} \\ \bar{p}'_{o,5} \\ \bar{p}'_{o,6} \\ \bar{p}'_{o,7} \\ \bar{p}'_{o,8} \end{bmatrix} = \begin{bmatrix} 24/64 \\ 6/64 \\ 34/64 \\ 24/64 \\ 2/64 \\ 132/64 \\ 24/64 \\ 30/64 \end{bmatrix} + \left[\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 48/64 & 12/64 & 4/64 & -16/64 & 4/64 & 8/64 & -16/64 & -4/64 \\ 12/64 & 39/64 & 13/64 & 12/64 & 13/64 & -6/64 & 12/64 & -13/64 \\ 4/64 & 13/64 & 47/64 & 4/64 & -17/64 & -2/64 & 4/64 & 17/64 \\ -16/64 & 12/64 & 4/64 & 48/64 & 4/64 & 8/64 & -16/64 & -4/64 \\ 4/64 & 13/64 & -17/64 & 4/64 & 47/64 & -2/64 & 4/64 & 17/64 \\ 8/64 & -6/64 & -2/64 & 8/64 & -2/64 & 60/64 & 8/64 & 2/64 \\ -16/64 & 12/64 & 4/64 & -16/64 & 4/64 & 4/64 & 48/64 & -4/64 \\ -4/64 & -13/64 & 17/64 & -4/64 & 17/64 & 2/64 & -4/64 & 47/64 \end{bmatrix} \right] \cdot \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{bmatrix} \qquad \text{(I.B.2n)}$$

For (I.B.21) we get:

$$\bar{\underline{p}}'_u = \min_{\underline{r}} \|\bar{\underline{p}}'_o\|_2 = \bar{A}^+ \cdot \underline{g} =$$

$$\begin{bmatrix} \bar{p}'_{u,1} \\ \bar{p}'_{u,2} \\ \bar{p}'_{u,3} \\ \bar{p}'_{u,4} \\ \bar{p}'_{u,5} \\ \bar{p}'_{u,6} \\ \bar{p}'_{u,7} \\ \bar{p}'_{u,8} \end{bmatrix} = \begin{bmatrix} 48/64 & 12/64 & 4/64 & -16/64 & 4/64 & 8/64 \\ 12/64 & 39/64 & 13/64 & 12/64 & 13/64 & -6/64 \\ 4/64 & 13/64 & 47/64 & 4/64 & -17/64 & -2/64 \\ -16/64 & 12/64 & 4/64 & 48/64 & 4/64 & 8/64 \\ 4/64 & 13/64 & -17/64 & 4/64 & 47/64 & -2/64 \\ 8/64 & -6/64 & -2/64 & 8/64 & -2/64 & 60/94 \\ -16/64 & 12/64 & 4/64 & -16/64 & 4/64 & 8/64 \\ -4/64 & -13/64 & 17/64 & -4/64 & 17/64 & 2/64 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1/2 \\ 9/4 \end{bmatrix} = \begin{bmatrix} 24/64 \\ 6/64 \\ 34/64 \\ 24/64 \\ 2/64 \\ 132/64 \\ 24/64 \\ 30/64 \end{bmatrix} \qquad \text{(I.B.2o)}$$

From (I.B.2o), using the definitions of relative preferences and expressing them all in terms of marginal preference with respect to the instrumental price variable p_t, we derive:

$$\forall^{t=1,\ldots,T}$$

$$\frac{\overset{*}{\omega}_{y'_{R,t}}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)}{\overset{*}{\omega}_{p_t}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)} = \frac{\bar{p}'_{u,1}}{\bar{p}'_{u,6}} = \frac{12}{66}$$

$$\frac{\overset{*}{\omega}_{y_{p,t}}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)}{\overset{*}{\omega}_{p_t}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)} = \frac{\bar{p}'_{u,2}}{\bar{p}'_{u,6}} = \frac{3}{66}$$

$$\frac{\overset{*}{\omega}_{s_{u,t}}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)}{\overset{*}{\omega}_{p_t}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)} = \frac{\bar{p}'_{u,3}}{\bar{p}'_{u,6}} = \frac{17}{66}$$

(I.B.2p)

$$\frac{\overset{*}{\omega}_{y_{L,t}}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)}{\overset{*}{\omega}_{p_t}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)} = \frac{\bar{p}'_{u,4}}{\bar{p}'_{u,6}} = \frac{12}{66}$$

$$\frac{\overset{*}{\omega}_{x_t}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)}{\overset{*}{\omega}_{p_t}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)} = \frac{\bar{p}'_{u,5}}{\bar{p}'_{u,6}} = \frac{1}{66}$$

$$\frac{\overset{*}{\omega}_{p_t}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)}{\overset{*}{\omega}_{p_t}(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t)} = \frac{\bar{p}'_{u,6}}{\bar{p}'_{u,6}} = 1$$

The <u>relative preference elasticities</u> corresponding with system (I.B.2p) can be calculated as follows:

$$\forall_t \; t = 1,\ldots,T$$

$$\frac{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t))}{E(y'_{R,t})}}{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},Y_{L,t},x_t,p_t))}{E(p_t)}} = \frac{12}{66} \cdot \frac{y'_{R,t}}{p_t} \qquad \text{(a')}$$

$$\frac{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t}x_t,p_t))}{E(y_{p,t})}}{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t))}{E(p_t)}} = \frac{3}{66} \cdot \frac{y_{p,t}}{p_t} \qquad \text{(b')}$$

$$\frac{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t))}{E(s_{u,t})}}{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t))}{E(p_t)}} = \frac{17}{66} \cdot \frac{s_{u,t}}{p_t} \qquad \text{(c')}$$

(I.B.2q)

$$\frac{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t))}{E(y_{L,t})}}{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t))}{E(p_t)}} = \frac{12}{66} \cdot \frac{y_{L,t}}{p_t} \qquad \text{(d')}$$

$$\frac{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t))}{E(x_t)}}{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t))}{E(p_t)}} = \frac{1}{66} \cdot \frac{x_t}{p_t} \qquad \text{(e')}$$

$$\frac{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t))}{E(p_t)}}{\dfrac{E(\omega_t(y'_{R,t},y_{p,t},s_{u,t},y_{L,t},x_t,p_t))}{E(p_t)}} = 1 \cdot \frac{p_t}{p_t} = 1 \qquad \text{(f')}$$

If we know the observed values $y'_{R,t}, y_{p,t}, s_{u,t}, y_{L,t}, x_t$ and p_t $\forall_t^{t=1,\ldots,T}$ conceiving them to be optimal, we can calculate the numerical values of the relative preference elasticities during the years of the observation horizon consisting of T years by the use of system (I.B.2q). In our cases I and II indicated below, the observed average optimal values of the target and instrumental policy-variables are generated by the "CS"-model discussed in the preceding paragraph. However, in this example we do not compare the relative preference elasticity ratios for every year belonging to the same observation horizon, but now we want to compare the relative preference elasticity ratios corresponding to different observation horizons, where the first three horizons are characterized by the average results over the business-cycle time-track of the relevant policy-variables, denoted as Trend Cases 1, 2 and 3 in the tables I.B.1.1/ I.B.1.3 of the foregoing paragraph. The results are summarized as case I hereafter. The other three horizons are again characterized by mean business-cycle results of which the first ones are those of Trend Case I in table I.B.1.1, the second horizon is a combination of this latter Trend Case and Trend Case 2 in table I.B.1.2, whereas the third horizon has been composed by adding up the trend results for the sole relevant policy variables as given in each of the aforementioned tables I.B.1.1/ I.B.1.3. The final results are summarized as Case II here below.
Actually, every horizon consists of infinitely repeated short-term business-cycles of 6 periods. The averages of the short-term business-cycle results constitute trendvalues, being average deviations from the original structural values as explained in the preceding paragraph. For the only reason as we discussed at the end of this same paragraph the sign of the results listed in the tables I.B.2.1 and I.B.2.3 has to be inverted if pushes and pulls had been valued in a negative way keeping them unchanged with respect to their original absolute magnitude.

Case I: Tables I.B.2.1 and I.B.2.2:

Table I.B.2.1.

Horizon $\bar{t}$ / Policy variables	Trend Case of Table I.B.1.1 $\bar{t}$ = 1	Trend Case of Table I.B.1.2 $\bar{t}$ = 2	Trend Case of Table I.B.1.3 $\bar{t}$ = 3
$\bar{y}'_{R,\bar{t}}$	0	0	0
$\bar{y}_{p,\bar{t}}$	-0.67	+ 0.50	-6.25
$\bar{s}_{u,\bar{t}}$	-0.23	- 9.50	-2.75
$\bar{y}_{L,\bar{t}}$	-0.57	+ 1.50	-5.25
$\bar{x}_{\bar{t}}$	-0.44	+10.00	-3.50
$\bar{p}_{\bar{t}}$	0.20	+ 2.00	+2.00

From system (I.B.2q) and Table I.B.2.1 we derive Table I.B.2.2:

Table I.B.2.2:

Horizon $\bar{t}$ / relative preference elasticities	$\bar{t}$ = 1	$\bar{t}$ = 2	$\bar{t}$ = 3
(a')	0	0	0
(b')	- 422.1/2772	+ 31.5/2772	- 393.75/2772
(c')	- 821.1/2772	-3391.5/2772	- 981.75/2772
(d')	-1436.4/2772	+ 378 /2772	-1323 /2772
(e')	- 92.4/2772	+ 210 /2772	- 73.5 /2772
(f')	1	1	1
Σ(a') - (f')	0	0	0

Case II: Tables I.B.2.3 and I.B.2.4

Table I.B.2.3

Policy variables \ Horizon $\bar{t}$	$\bar{t}$ = 1	$\bar{t}$ = 1+2	$\bar{t}$ = 1+2+3
$\underline{\bar{y}}_{R,\bar{t}}$	0	0	0
$\underline{y}_{p,\bar{t}}$	-0.67	-0.17	- 6.42
$\underline{s}_{u,\bar{t}}$	-0.23	-9.73	-12.48
$\underline{y}_{L,\bar{t}}$	-0.57	+0.93	- 4.32
$\underline{x}_{\bar{t}}$	-0.44	+9.56	+ 6.06
$\underline{p}_{\bar{t}}$	0.20	+2.20	+ 4.20

From system (I.B.2q) and Table I.B.2.3 we derive Table I.B.2.4:

Table I.B.2.4:

relative preference elasticities \ Horizon $\bar{t}$	$\bar{t}$ = 1	$\bar{t}$ = 1+2	$\bar{t}$ = 1+2+3
(a')	0	0	0
(b')	- 422.1/2772	- 9.73/2772	- 192.6/2772
(c')	- 821.1/2772	-3157.82/2772	-2121.6/2772
(d')	-1436.4/2772	+ 213.05/2772	- 518.4/2772
(e')	- 92.4/2772	+ 182.05/2772	+ 60.6/2772
(f')	1	1	1
Σ(a') - (f')	0	0	0

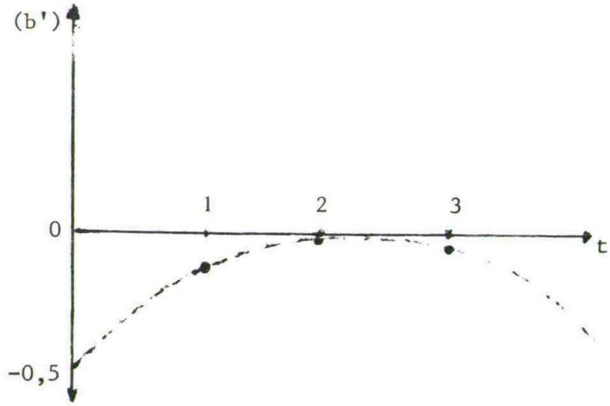

Figure I.B

Investigation on stability through time of the relative preferences or more likely of the relative preference elasticities as expressed in the above tables turns out to be not so very difficult. The nature of the evolution over time of the few relative preference elasticity data with regard to any target or instrumental variable becomes clear immediately. Therefore it may not be worthwhile to test stability of the preference-structure, for instance by the use of orthogonal polynomial fitting in order to find a polynomial of low degree that "adequately" describes the data.[6)] In this exercise we are finally dealing with three observation or determination periods ($\bar{t}$=1, = 1+2, = 1+2+3), so a compromise between the requirement for simplicity and the desire to get a good fit needs not to be made here. Otherwise, in extensive data-cases we aim at a low degree polynomial which satisfies the conditions of simplicity and of being a "good" fit. The ultimate compromise to be made can be facilitated by the use of additional assumptions with regard to the distribution of the sum of squares of residuals (true errors) of the polynomials to be fitted. Such particular assumptions may be imply for instance that the student-t distribution can be used to fix confidence intervals for the partial regression coefficients of a fitted polynomial of second or higher degree. There are many other possibilities to make the ultimate choice which we shall not discuss now.

But for the sake of completeness of our attempt to demonstrate our line of approach with regard to the empirical investigations (using the DSID-application model)[7)], we shall fit below a linear and a quadratic poly-

nomial in order to find a best "fit" with respect to the relative preference elasticity data of an arbitrary policy variable.
For this special case we will investigate the data concerning the Gross National Production variable y_p. These data are stated in the (b')-row of Table I.B.2.4. The corresponding relevant polynomials are recorded in Table I.B.2.5 whereas Table I.B.2.6 reviews the Analysis of Variance of the relative preference elasticity data. This latter table tells us that the linear polynomial does not describe the data adequately.
Of course, in this special case the quadratic polynomial fits the data perfectly. In general every (n-1)-degree polynomial always describes the data (with respect to n observations) perfectly and will go through any point.

Table I.B.2.5

Polynomials for Relative Preference Elasticity Data b' of Table I.B.2.4				
$\bar{t}' = i$ (horizon coded)	1	2	3	Σp^2
(b_i')	$\frac{-442.1}{2772} = -0.152$	$\frac{-9.73}{2772} = -0.003$	$\frac{-192.6}{2772} = -0.069$	
p_1	-1	0	1	2
p_2	1	-2	1	6
$\bar{b}' = \frac{\Sigma b_i'}{n} = -0.075$	$\Sigma(b_i')^2 = 0.028$			
$n = 3$	$\Sigma p_1 b_i' = 0.082$		$\Sigma p_2 b_i' = -0.214$	

Table I.B.2.6

Variance Analysis of Relative Preference Elasticity Data of Real Gross National Product with respect to the Internal General Price-level					
Variation	Degrees of Freedom	Sum Squares	Mean Squares	F-value $\alpha = 0.05$	P
Total	3	0.028			
Reduction for Mean	1	0.0169			
Remainder for Mean	2	0.0111			
Linear	1	0.0034	0.0034	< 1	> 5%
Error for Linear	1	0.0077	0.0077		
Quadratic	1	0.0077	0.0077		

Estimation of the parameters in the quadratic polynomial can be performed by the use of orthogonal polynomial fitting because we are dealing with equally spaced $x = \bar{t}'$ ($\bar{t}' = 1,2,3$) in the curvilinear model:

$$b' = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + e \qquad \text{(I.B.2r)}$$

For (I.B.2r) we derived:

$$b' = -0{,}515 + 0{,}47\ t - 0{,}107\ t^2 \qquad \text{(I.B.2s)}$$

It is easily verified that equation (I.B.2s) is a concave function generating perfectly the original DSID-model results with regard to the relative preference elasticity of the Real Gross National Product-variable for $t = \bar{t} = 1$, 1+2 and 1+2+3 as stated in Table I.B.2.4. Its graphical shape has been represented in Figure I.B.

Par. I.B.3. Evaluation, Notes and References

Apart from a short evaluation of the results already obtained in the foregoing paragraphs, we want to use these concluding remarks for the derivation of some other results that may tell strongly in favour of our DSID-approach to the measurement of stability of the revealed preferences of policy makers.

The first purpose of the paper at issue was to show within the limits of an exercise in the laboratory of ex ante model building, in what way the DSID-method delivers an acceptable ex post measure of the preference structure of a policy decision unit. The implicit values of the relative preference elasticities and especially their evolution through time have to tell us that, if this policy decision unit took its decisions in correspondence with the scheme of optimal economic policy, what would have been the stability or instability of its intertemporal preference structure. In the latter sense it may give a better insight in the existing instability in actual political relations if the DSID-application model is applied for empirical investigation of the real world economic-political outcomes. The present exercise denotes the critical border lines within which one should compare ex ante political intentions with ex post revealed preferences. Thus we might deduce something about the efficiency and consistency of political activity. Our proposal of investigating social preferences may perhaps contribute to a more profound explication of the real causes of long run and short run economic waves. These causes probably are strongly related to an alternating dominance in the course of time of different powerblocks to be distinguished inside a policy decision unit. Interdependence of the outcomes of such interrelated social-economic and political processes and the question about their desirability should therefore always be accepted as legitimate reasons in justification of those 'practical' welfare research-workers who are perhaps fighting windmills.

To what extent we are 'beating the air' will be left to the reader of the present paper.

Let us now evaluate the main results:

(1) In Par. I.B.1 we established the 'CS'-model in order to isolate in a correct numerical way the true economic-political measures of a policy decision unit. Apart from the autonomous variables with regard to pushes and pulls, the concrete structure of the model can be qualified as a system of 'monopoly capitalism'. The ignorance of the market clearing function of prices on the product market and the assumption of a less imperfect labour market regime refer to a dominant powerblock of employers inside the policy decision unit.
If we accept for a moment the pure hypothetical assumption of the a priori knowledge of our 'CS'-model by the policy decision unit, the mentioned powerblock of entrepreneurs would perhaps never accept the consequences of the economic-political measures listed in the tables I.B.1.1/I.B.1.3. Therefore, let us assume their wage policy, spending policy and price policy measures were opposite to those that were analyzed in Paragraph I.B.1. In consequence of the symmetrical nature of the CS-model, all the simulated results would have got the inverted sign, and by this the original negative conclusions on page 15 would become positive ones. If we now turn our attention to the demonstration of the DSID-model procedure in Paragraph I.B.2, we can immediately verify that it takes care of the insensibility to the aforementioned opposite simulation situations of the relative preference (elasticity) systems of (I.B.2.p) and (I.B.2.q) respectively. Therefore we may conclude that the preference structure, and especially its stability through time to be revealed ex post by the DSID-model only depends on the explicitely known coefficients of the ex post assumed side conditions of the optimal quantitative economic policy scheme of (I.B.2.a), in combination with the calculated ratios of the observed outcomes with regard to the target and instrumental variables. Within the theoretical frame-work of the present exercise these outcomes were generated by the 'CS'-model and by this latter circumstance the ex post revealed preference structure can be compared with ex ante policy measures (= political intentions). It is worthwile to note again that the reversibility of the 'CS'-model results appears not to touch the series of ratio-values and thus it lets the preference structure to be revealed unchanged if all the policy-measures (pushes and pulls) are valued in a unique way.

(2) From the two main cases I and II of which the numerical results have been listed in the tables I.B.2.1/I.B.2.4, we immediately derive that all the six (five different) situations show values for the relative preference elasticities which all together sum up to zero. The latter result can easily be understood in mathematical terms. If we denote the matrix of coefficients appearing in the side-conditions of I.B.2b by B_t, its Null-space by $N(B_t)$ and the range of its transpose by $R(B_t^*)$, than it can be verified that vectors of the observed optimal target and instrumental values are elements of $N(B_t)$. On the other hand, the first six elements of the $\bar{\underline{P}}'_u$-vector of system I.B.2o composes a vector that belongs to the 'Range' space of the linear transformation B_t^*, i.e., is an element of $R(B_t^*)$; $N(B_t)$ and $R(B_t^*)$ are the orthogonal complements of each other and by this the aforementioned two vectors are perpendicular in the Euclidean space R^6. By this their multiplication satisfies the condition
$R(B_t^*).N(B_t) = 0.$
In 'welfare'-theoretical terms the sum-up-to-zero result must be interpreted as a situation wherein the policy decision unit aims at policy measures in the course of time which stabilize the optimal welfare level in deviation of its equilibrium value on the path of exponential growth. As we saw in paragraph I.B.1, this latter path should be considered as the initial situation to be referred to for the present evaluation.
If we consider again the tables I.B.2.2 and I.B.2.4, we derive for the observed horizon of $\bar{t} = 1$ that the ex ante wage policy measure is translated into the revealed preference structure, saying that a positive marginal deviation in the internal price level must be compensated by positive marginal deviations in the values of the other target and instrumental variables and vice versa. To what extent the aforestated 'welfare'-condition is statisfied by the individual contribution of the different variables, is indicated by the 'trade-off'-values listed in the $(\bar{t} = 1)$-columns of the two tables.
Further investigation on how other ex ante policy measures and combinations of them are translated into revealed preference structures, can be based on the data from the other columns of the two tables.

A demonstration of such thorough investigation on the preference structure as revealed by the aforementioned 'trade-off' values in consequence of wage policy can tell us the following things about the view of the policy decision authorities:

a) A 1% extra increase of the internal price level relative to its calculated new trendvalue (as listed in the tables I.B.2.1. and I.B.2.3.)is offset by an analogue overall -1%- decrease of the other instrumental and target variables and vice versa. This is true for all of them other than the profit income- -share-variable. We do not calculate the trade-off between the latter variable and the other ones as the coefficients are zero or not sensible.

b) In the present case of the ex ante known wage-policy measure it is sensible to calculate the implicit trade-offs indicating, Ceteris Paribus, which changes in the targets and instruments compensate for an extra decrease of 1% of the instrumental labour income value. For horizon $\bar{t} = 1$, it can be derived from the tables I.B.2.2. and I.B.2.4. that these trade-off values are -3.40, -1.75 , -15.54 and +0.51. They reflect marginal ameliorations of 3.4%, 1.75% and 15.54% with regard to the national production level, the balance of exports and imports and the level of national spendings respectively, whereas the price level should be rised by over 0.5%.
These latter results correspond very closely with the revealed preference structure in consequence of the ex ante known price policy measure (see table I.B.2.2. for $\bar{t} = 3$). In summary we may conclude that wage and price policy are revealed by the same preference structure to be qualified as a consistent result in the present case of monopoly capitalism!

The revealed preference structure in consequence of the ex ante known spending-policy measure (see table I.B.2.2. for $\bar{t} = 2$) indicates a positive overall-trade-off of the policy variables with regard to the target variable of the balance of exports and imports. An extra 1%-decrease of the latter target value is offset by an extra 1%-increase of the other variables (apart from the profit income-share variable). This latter result can be seen again as consistent with the situation of monopoly capitalism. The same is true for the revealed preference structures of combinations of the three distinct policy measures as can be derived from Table I.B.2.4. for $\bar{t} = 1 + 2$ and $\bar{t} = 1 + 2 + 3$.

Only for the sake of completeness, we investigated on stability through time for the individual trade-off coefficient with regard to the National Product variable.

For the sake of shortness, the various conclusions to be drawn at this stage, will be left to the reader.

(3) In technical sense the DSID-model has been strongly based on the synthesis of the Lagrange and Moore-Penrose inverse techniques. This way of doing can be justified on three grounds:

a) It selects a unique vector of some relative preferences whatever the actual value of the 'numéraire' $\lambda_n \neq 0$ may be.
In the present exercise $\lambda_{3,t} = 1$ implies that the marginal preferences with regard to the target variables all together sum up to 1 as can be verified from I.B.2o. By this it will be reasonable to assume that the condition $\lambda_n \neq 0$ has been satisfied.

b) The DSID-application model ultimately wants to test stability through time of the relative preference elasticities. Orthogonal polynomial fitting and other statistical tests can be based on solutions which are very sensitive to variations in the known parameters of the DSID-system which biases them against the H_0-hypothesis of invariability.

c) It uses Graybill's computing formula for the Moore-Penrose inverse that appears to be the best one in terms of accuracy and speed of computation. (We refer to note 5. hereafter.)

(4) The present exercise still leaves a number of problems unsolved. Some of them will be solved in the second paper that we promised in the introduction paragraph.

In the present paper we showed how the DSID-approach takes care of the situation wherein the results are not influenced anymore by the a priori functional form of the preference function. Moreover, the problem of the second-order conditions have been side-stepped by the introduction of a peculiar concept of the relative preference (elasticity). All this will be admissible if one can prove that there always exists an objective function that will be maximized by a policy decision unit. This latter proof has been given in the Appendix A.

Notes

1. See the bibliographical references, numbers 5 and 6.

2. A good overview of these problems are stipulated in the bibliographical references, numbers 7 and 8.

3. Referring to the introduction and to the bibliographical reference, number 9, it becomes clear that we are dealing with a modest form of monopoly capitalism.

4. The exact relationship can be expressed as:

$$\Delta x_t = \left\{\frac{\tilde{x}_t - \tilde{x}_{t-1}}{\tilde{x}_{t-1}} - \frac{\tilde{x}_{0_t} - \tilde{x}_{0_{t-1}}}{\tilde{x}_{0_{t-1}}}\right\} \left\{\frac{100 + x_{t-1}}{\tilde{x}_{0_t} / \tilde{x}_{0_{t-1}}}\right\}$$

5. We refer to the bibliographical references, numbers 2, 3, 5 and 6. The procedure which we used, has been based on the same Graybill's computing formula for a g-inverse that appeared to be the best one in terms of accuracy and speed of computation from the point of view of Kymn c.s.

6. See the bibliographical references, numbers 1 and 4.

7. See the introduction paragraph.

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Appendix A: Proof of existence of a global maximum of an objective function of the optimizing problem of quantitative economic policy and its inverse.

Establishing the theoretical and the DSID-application-model for derivation of the trends in the relative preference elasticities, considering them as indicative of the evolution of implicit social preferences, we suggested problems of the second-order conditions could be side-stepped (see par. I.B.2 and the bibliographical references, numbers 5 and 6). Hereafter we will give the proof of existence of the global maximum of an objective function of the postulated optimizing problem of quantitative economic policy and its inverse. The proof boils down to show under what circumstances ex post satisfaction of the first order Lagrange-conditions is necessary and sufficient to an objective function will be maximized, without bothering about its functional form.

Proof: Suppose a maximizing problem of mathematical programming can be formalized as follows:

$$\max/\omega(y_j;z_k)/ \tag{A.a}$$

where $j = 1,\dots,J;\quad k = 1,\dots,K$ and $(y_j,z_k) \in W'$;

where $W' = \{(y_j,z_k) \in R^{J+K},\ f_l(y_j,z_k,v_i) = 0\}$

where $i = 1,\dots,I$

$l = 1,\dots,N$

$N = J + I$

y_j $(j=1,\dots,J)$, z_k $(k=1,\dots,K)$ and v_i $(i = 1,\dots,I)$

denote respectively the target, instrumental and irrelevant endogenous variables of the economic model $f_l\ (y_j,z_k,v_i) = 0$ $(l = 1,\dots,N)$.

Let $\underline{x} = (y_j,z_k,v_i)$

$\underline{f}(\underline{x}) = \underline{0} \qquad \underline{f} : R^{N+K} \rightarrow R^{N}$

$R^{N+K} \supset W = \{\underline{x}/\underline{f}(\underline{x}) = \underline{0}\}$ (A.b)

$\Omega(\underline{f},\omega)$ the set of optimal solutions.

Acceptance of the ex post approach, we are dealing with the inverse of problem (A.a), i.e., the realized values of the variables y_j, z_k and v_i are known and conceived of as results of scheme (A.a); so they are optimal values:

$\underline{f}(\underline{x}_0) = \underline{0} \qquad \underline{x}_0 \in \Omega(\underline{f},\omega)$.

Let

(1) $\underline{f}$ be continuously differentiable in the neighbourhood of $\underline{x}_0$,

(2) $r(D\underline{f}\ (\underline{x}_0)) = N$; i.e., the functional matrix

$D\ \underline{f}(\underline{x}_0)$ is full-ranked.

If (1) and (2) are satisfied, we can say:

there exists a neighbourhood U of $\underline{0} \in R^{N+K}$
and a neigbourhood V of $\underline{x}_0 \in R^{N+K}$ for which there exists a diffeomorfism (bijection) g:

$g : U \rightarrow V$

so that

(1') $g(\underline{0}) = \underline{x}_0$

(2') $\forall_{\underline{x}} \in V \cap W : g^{-1}(\underline{x}) = (P_1,\dots,P_K,0_1,\dots,0_N)$

where 0_i $(i = 1,\dots,N)$ are null-valued scalars, so $\underline{x} = g(P_1,\ldots,P_K,\ 0_1,\ldots,0_N)$.

If $\underline{x} \in V \cap W$ there exists a function h:

$$h : R^{N+K} \sim \rightarrow R^K$$

$$h(P_1,\ldots,P_K,\ 0_1,\ldots,0_N) = (P_1,\ldots,P_K)$$

$$h^{-1}(P_1,\ldots,P_K) = (P_1,\ldots,P_K,\ 0_1,\ldots,0_N)$$

$$h \circ g^{-1} : V \cap W \rightarrow R^K,$$

with

$$h \circ g^{-1}(\underline{x}) = (P_1,\ldots,P_K)$$

and

$$R^K \rightarrow V \cap W$$

$$g \circ h^{-1} : R^K \rightarrow V \cap W$$

Example: Suppose a maximizing problem of mathematical programming can be formalized as (A.a) with j = 1,2,; k = 1,2 and i = 1,2.

Instead of (A.a) we derive (A.c):

$$\max/\omega(y_1,y_2,z_1,z_2)/ \tag{A.c}$$

where $(y_1,y_2,z_1,z_2) \in W'$

$$W' = \{(y_1,y_2,z_1,z_2)/(y_1,y_2,z_1,z_2) \in R^4,\ f_1(y_1,y_2,z_1,z_2,v_1,v_2) = 0\}$$

$(1 = 1,\ldots,4)$
$(N = J+I = 4)$

Instead of (A.b) we derive (A.d):

Let $\underline{x} = (y_1,y_2,z_1,z_2,v_1,v_2)$

$$\underline{f}(\underline{x}) = \underline{0} \qquad\qquad \underline{f} : R^6 \to R^4 \qquad\qquad (A.d)$$

$$R^6 \supset W = \{\underline{x}/f(\underline{x}) = 0\} .$$

In the ex post approach we are dealing with the inverse of problem (A.c), i.e., the realized values of the variables y_1, y_2, z_1, z_2, v_1 and v_2 are known and conceived of as results of scheme (A.c); so they are optimal values:

$$\underline{f}(\underline{x}_0) = \underline{0} \qquad\qquad \underline{x}_0 \in \Omega(\underline{f},\omega) .$$

Let

(1) $\underline{f}$ be continuously differentiable in the neighbourhood of $\underline{x}_0$;

(2) r (D $\underline{f}(\underline{x}_0)$) = 4; i.e., the functional matrix D $\underline{f}(\underline{x}_0)$ is full-ranked.

If (1) and (2) are satisfied, we can say:

there exists a neighbourhood V of $\underline{x}_0 \in R^6$ and a neighbourhood U of $\underline{0} \in R^6$ for which there exists a bijection g:

$$g : U \to V$$

so that

$$(1') \; g(\underline{0}) = \underline{x}_0$$

$$(2') \; \forall \underline{x} \in V \cap W : g^{-1}(\underline{x}) = (P_1,P_2,0,0,0,0),$$

so

$$\underline{x} = g(P_1,P_2,0,0,0,0).$$

If $\underline{x} \in V \cap W$, there exists a function h:

$$h : R^6 \sim\to R^2$$

$$h(P_1,P_2,0,0,0,0) = (P_1,P_2)$$

$$h^{-1}(P_1,P_2) = (P_1,P_2,0,0,0,0)$$

$$h \circ g^{-1} : V \cap W \to R^2$$

with

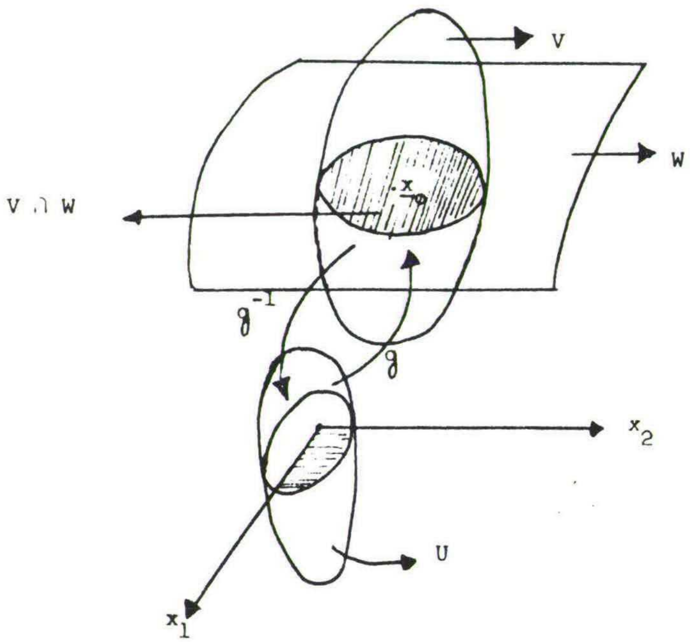
$$h \circ g^{-1}(\underline{x}) = (P_1, P_2)$$

and

$$R^2 \to V \cap W$$

$$g \circ h^{-1} : R^2 \to V \cap W.$$

Graphical presentation of the example:



$$R^6 \supset W = \{ \underline{x} \,/\, f(\underline{x}) = 0 \}$$

$$R^6 \supset V$$

$$R^6 \supset V \cap W \ni \underline{x}$$

$$R^6 \supset U$$

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